Grade Level/Course: Algebra 1 & Algebra 2
Lesson/Unit Plan Name: Introduction to Inverse Functions
Rationale/Lesson Abstract: This lesson is designed to introduce inverse functions by building on students' prior knowledge of functions and graphing functions. It is intended for students to not only see the two variables exchange places in the equation, but also to highlight how the characteristics of the two graphs relate & vary based on this relationship. Students should have a firm grasp of graphing a linear function as well as writing the equation of a line in slope intercept form.
Timeframe: One period
Common Core Standard(s): F.BF.4 – Find Inverse Functions

Instructional Resources/Materials:
Warm up, student note-taking guide, graph paper and pencil

Activity/Lesson:

Warm Up Solutions:

Answers A, C, and D are relations that are functions. Use this problem to remind students about the vertical line test and why the test works – If a vertical line hits two or more points, then the relation has an input that provides two or more outputs and is not a function.	Domain: -3, 0, 1, 2, 5 Range: -3, 1, 4, 8
a.) Function – A relation which associates every element in one set (the domain) with one and only one element in another set (the range). b.) Yes. Every element in the domain has only one element in the range.	$\frac{1}{2}y-7=x$ $\frac{1}{2}y-7+7=x+7$ $\frac{1}{2}y=x+7$ $\frac{1}{2}y+\frac{1}{2}y=(x+7)+(x+7)$ $y=2x+14$

Introduction to Inverse Functions:

Pass out student note-taking guide and *begin by defining the inverse of a function:*

"*The inverse of a function is the relation formed when the independent variable is exchanged with the dependent variable of a given relation***."**

Give this example to make sure students understand the definition of an inverse.

<u>Example 1:</u>
Find the inverse of the following function:
$f:\{(3,4),(2,8),(-1,7),(0,-3)\}$

Answer: $f^{-1}:\{(4,3),(8,2),(7,-1),(-3,0)\}$
Let students know that f^{-1} is called "the inverse of f " or " f inverse" and also let them know that the -1 is not an exponent.

Example 2:

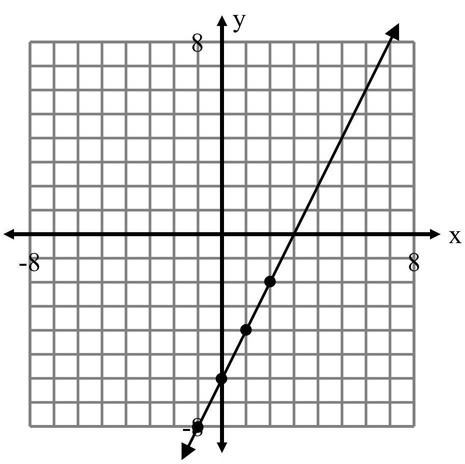
Graph the function and its inverse. Then find the equation of the inverse function.

$f(x) = 2x - 6$

x	y
-1	-8
0	-6
1	-4
2	-2

$\Delta x = +1$ $\Delta y = +2$

$$m = \frac{\Delta y}{\Delta x} = \frac{+2}{+1} = 2$$



Have students fill out the table for the original function with the following inputs at left and graph the points on the graph as you go. Then remind students that the slope can be seen from the table by finding the ratio of the change in y over the respective change in x.

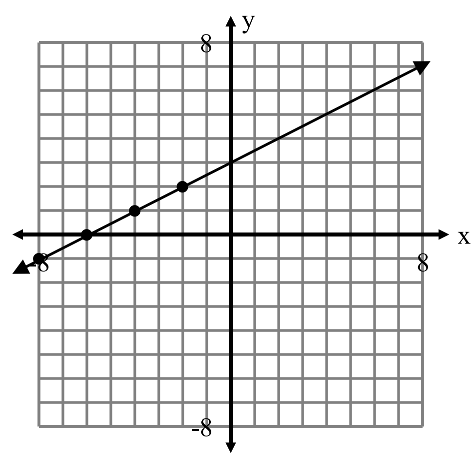
Then have students exchange the x and y coordinates of your table(left) to form points on your inverse function(table on right) just like example 1. Have the students graph these points as well and then draw the inverse function. Continue the pattern in the table and on the graph to find the y-intecept of the inverse function and then write the equation of the inverse function.

$f^{-1}(x)$

x	y
-8	-1
-6	0
-4	1
-2	2

$\Delta x = +2$ $\Delta y = +1$

$$m = \frac{\Delta y}{\Delta x} = \frac{+1}{+2} = \frac{1}{2}$$



The *y*-intercept is $(0,3)$

$$\therefore b = 3$$

Inverse: $y = \frac{1}{2}x + 3$

$$f^{-1}(x) = \frac{1}{2}x + 3$$

Think-Pair-Share

Have the students compare the two graphs individually and look for any relationships/differences between the two. Then ask them to share with a partner. After a few minutes have students share out what they noticed. Write conjectures for all to see.

Example 2 continued:

Finding the Inverse Function Algebraically

$$f(x) = 2x - 6$$
$$y = 2x - 6$$
$$f^{-1}: \quad x = 2y - 6$$
$$x + 6 = 2y - 6 + 6$$
$$x + 6 = 2y$$
$$\frac{x+6}{2} = \frac{2y}{2}$$
$$\frac{x}{2} + \frac{6}{2} = y$$
$$\frac{1}{2}x + 3 = y$$
$$\therefore f^{-1}(x) = \frac{1}{2}x + 3$$

Before this step, ask a student to reread the definition of an inverse. Then exchange the two variables in the equation and let the students know that the solutions for the equation are now reversed. For example in the original function an x-value of 3 produces a y-value of 0 (3,0), while in the inverse relation a y-value of 3 produces an x-value of 0 (0,3). Solving for y as a function of x creates your inverse function.

You Try:

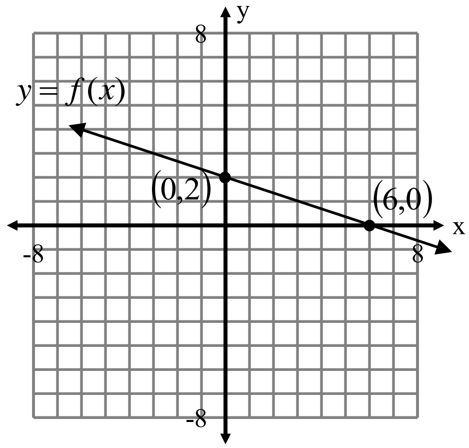
Find the inverse of the function $f(x) = -\frac{2}{3}x + 4$ **.**

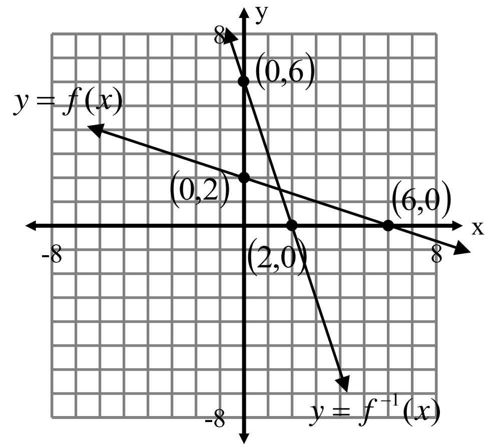
Walk around the room and find a student to debrief their work after giving everyone a few minutes to work it out. If another student did it graphically, have them show that as well and see if some of the relationships/differences discussed earlier during the Think-Pair-Share hold true.

$$f(x) = -\frac{2}{3}x + 4$$
$$y = -\frac{2}{3}x + 4$$
$$f^{-1}: \quad x = -\frac{2}{3}y + 4$$
$$x - 4 = -\frac{2}{3}y + 4 - 4$$
$$x - 4 = -\frac{2}{3}y$$
$$3(x-4) = 3\left(-\frac{2}{3}y\right)$$
$$3x - 12 = -2y$$
$$\frac{3x-12}{-2} = \frac{-2y}{-2}$$
$$-\frac{3}{2}x + 6 = y$$
$$\therefore f^{-1}(x) = -\frac{3}{2}x + 6$$

Example 3:
What is the inverse of the function whose graph is shown below?

- *Label the two points exaggerated on the graph of the function.*
- *Exchange their coordinates to find two points of the inverse function.*
- *Plot the two points and graph the inverse function by drawing the line through the two points.*
- *Then find the slope and y-intercept of the inverse to write the equation of the inverse function.*

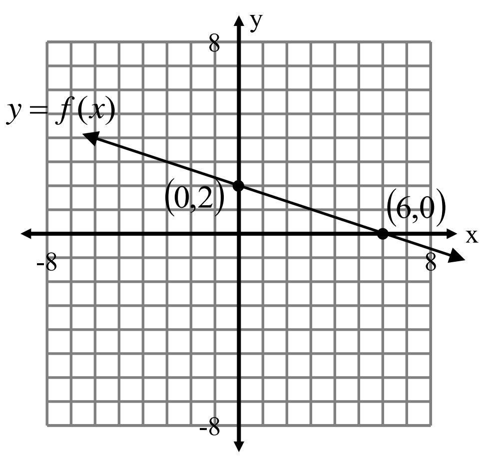




$$f^{-1}: \quad m = \frac{rise}{run} \qquad b = 6$$
$$= \frac{-6}{+2}$$
$$= -3$$

$$\boxed{\therefore f^{-1}(x) = -3x + 6}$$

Think-Pair-Share
Is there another way we could have used to find the inverse in example 3? Give the students a minute to think about it. Then ask them to share with a partner. Discuss as a class. Then have them verify that the inverse function is correct by finding the equation of $f(x)$ and then finding the inverse of f algebraically:



$$f: \quad m = \frac{rise}{run} \qquad b = 2$$
$$= \frac{-2}{+6}$$
$$= -\frac{1}{3}$$

$$\boxed{\therefore f(x) = -\frac{1}{3}x + 2}$$

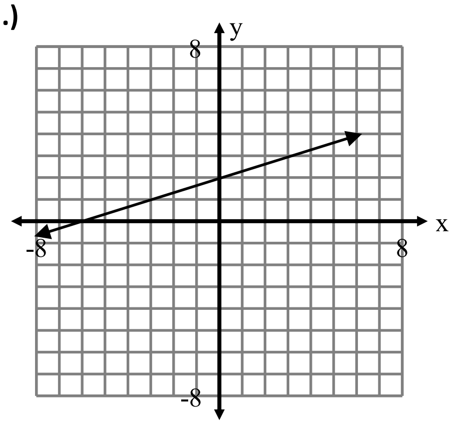
$$f(x) = -\frac{1}{3}x + 2$$
$$y = -\frac{1}{3}x + 2$$
$$f^{-1}: \quad x = -\frac{1}{3}y + 2$$
$$x - 2 = -\frac{1}{3}y + 2 - 2$$
$$x - 2 = -\frac{1}{3}x$$
$$-3(x-2) = -3\left(-\frac{1}{3}y\right)$$
$$-3(x-2) = y$$
$$-3x + 6 = y$$

$$\boxed{\therefore f^{-1}(x) = -3x + 6}$$

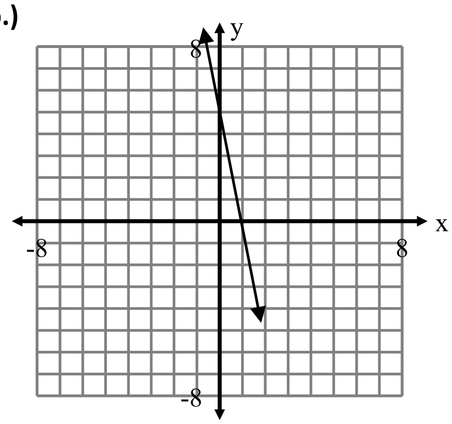
You Try

Draw the graph of the inverse function for each of the functions shown below on the same coordinate plane.

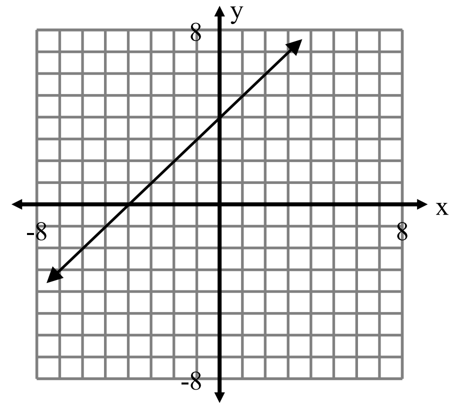
a.)

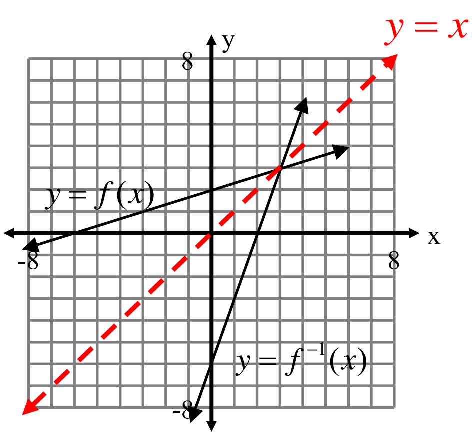


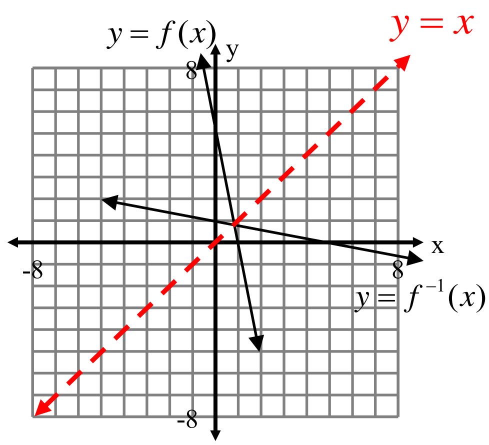
b.)

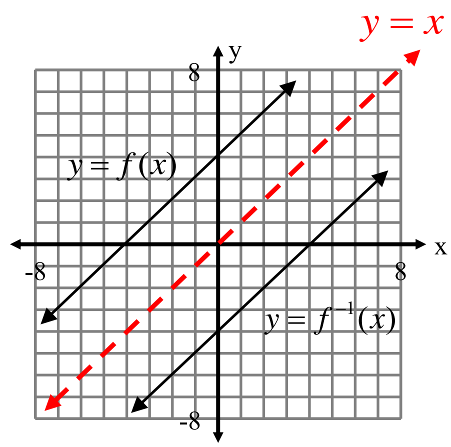


c.)









After giving the students a few minutes to finish the you try, draw the inverse functions on each of the graphs.

Draw the line $y = x$ on all three coordinate planes. Let the students know that "the inverse relation is a reflection of the original function across the line $y = x$". Introduce this relationship by focusing on **c.)** to explain what a reflection is and show how every point on the graph of the relation can be reflected across the line $y = x$ to form the graph of its inverse.

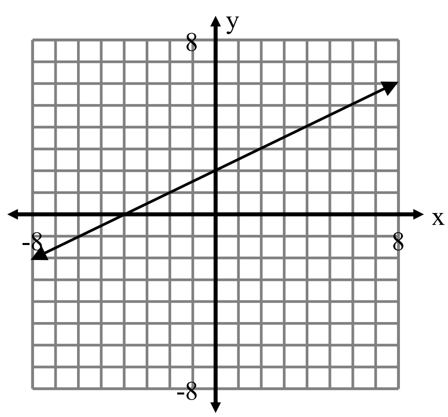
Assessment:

Have students think about what they learned today. Wait a minute then randomly select students to piece together a summary representative of the material discussed:

- The inverse of a function is formed when you exchange the independent and dependent variables of a given relation
- You can find the inverse function from the graph of a line by exchanging the *x* and *y* coordinates of the intercepts, plotting these new points, drawing the line and then writing the equation of the line.
- You can find the inverse from the equation of a function by replacing $f(x)$ with y, exchanging the *x* and *y* variables in the equation, solving for *y*, and then replacing *y* with $f^{-1}(x)$.

Exit Ticket –

1) Find the inverse of the function graphed below:



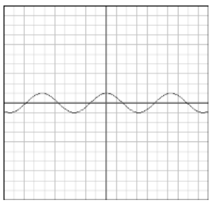
2) Find the inverse of the function $f(x)=\frac{1}{3}x+10$

Warm-Up

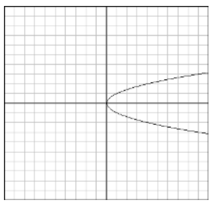
CST/CCSS: 18.0/F.IF.1

Select all the following relations that are functions:

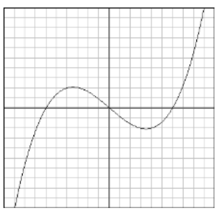
A)



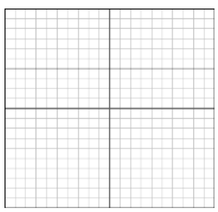
B)



C)



D)



Review: Algebra 1 F.IF.1

Find the domain and range of the relation:

$$\{(5,4),(0,4),(-3,1),(2,8),(1,-3)\}$$

Current: Algebra 1 F.IF.1

a) What is a function?

b) Is the relation in quadrant I a function?

Other: Algebra 1 A.CED.4

Solve for y.

$$\frac{1}{2}y - 7 = x$$

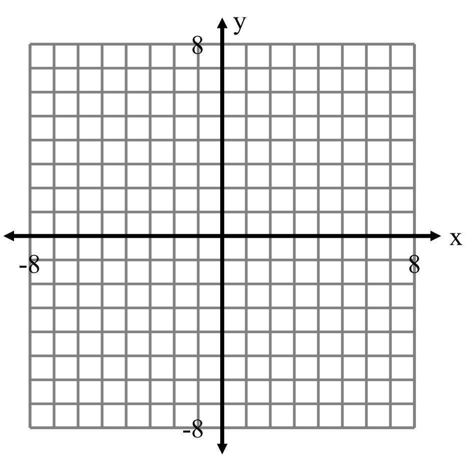
Definition of the inverse of a function:

Example 1: Find the inverse of the following function $f:\{(3,4),(2,8),(-1,7),(0,-3)\}$.

Example 2: Graph the function $f(x)=2x-6$ **and its inverse. Then find the equation of the inverse function.**

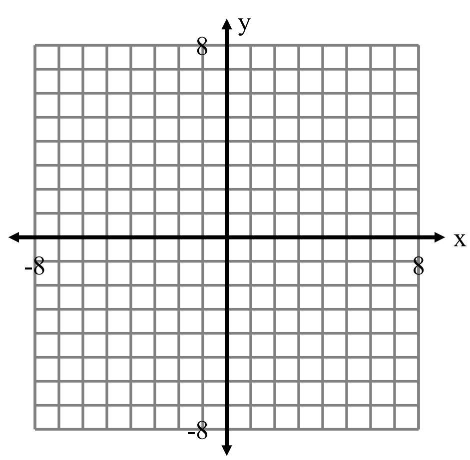
$f(x)=2x-6$

x	y



$f^{-1}(x)$

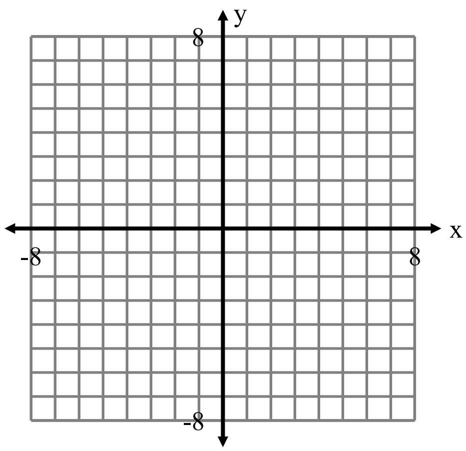
x	y



Finding the Inverse Function Algebraically

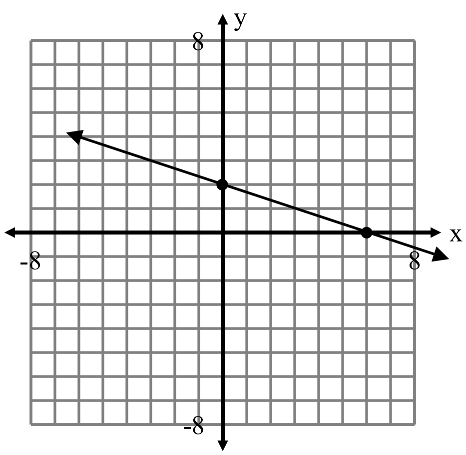
You Try:

Find the inverse of the function $f(x) = -\frac{2}{3}x + 4$**.**



Example 3:

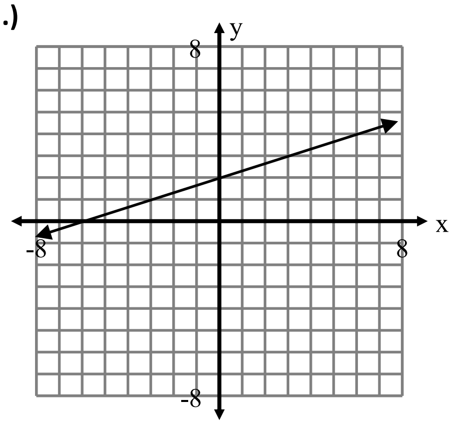
What is the inverse of the function whose graph is shown below?



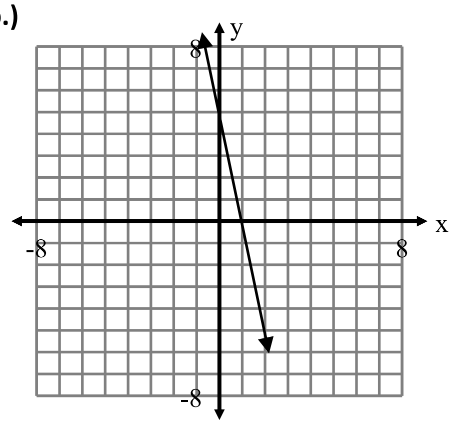
You Try

Draw the graph of the inverse function for each of the functions showed below on the same coordinate plane.

a.)



b.)



c.)